VGP352 – Week 3

- Agenda:
  - Quiz #1
  - BRDFs, part 1
    - Common ideas and terminology
    - Cook-Torrance BRDF
    - Micro-facet based BRDFs
  - Hand in assignment #1
  - Start assignment #2

BRDF

- Bi-directional reflectance distribution function
  - Notation is $f(\omega_o, \omega_i)$

“…describes the ratio of reflected radiance exiting from a surface in a particular direction (defined by the vector ω_o) to the irradiance incident on the surface from direction ω_i over a particular waveband.”

BRDF

- In English...
  - Given an arbitrary input direction, ω_i, and an arbitrary output direction, ω_o, we can the ratio of energy (light) transferred from ω_i to ω_o
- What does this tell us?

BRDF

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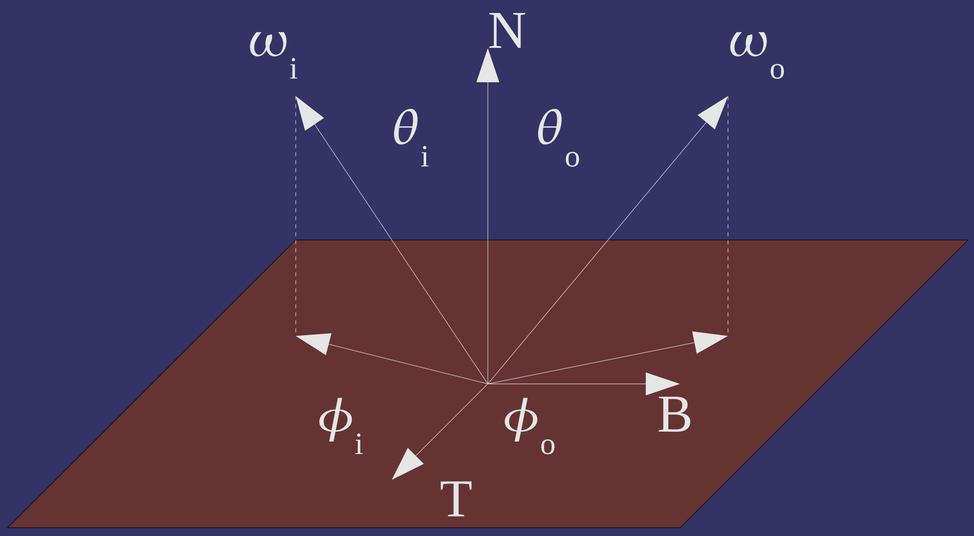
  - Given an arbitrary input direction, ω_i, and an arbitrary output direction, ω_o, we can the ratio of energy (light) transferred from ω_i to ω_o
- What does this tell us?
  - If we know where the light is coming from, we can calculate how much of the light is reflected in any direction
  - If we know a light reflection direction (i.e., viewing direction) we can calculate the contribution of every possible light input direction

BRDF

- ω consists of the two angles:
  - θ is the elevation angle, and it is measured relative to the surface normal
  - ϕ is the azimuth angle, and it is measured relative to the surface tangent



BRDFs for Lighting

- The amount of light reflected from a particular input vector to a particular output vector:

$$L(\omega_o) = f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i$$

Outgoing light intensity

A.k.a $N \bullet L$

Incoming light intensity

BRDFs for Lighting

- What if we want to calculate the amount light reflected to a particular output vector from *all possible* input vectors?

$$L(\omega_o) = \int_\Omega f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i \, d\omega_i$$

- What about this formulation seems broken?

BRDFs for Lighting

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$$L(\omega_o) = \int_{\Omega} f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i \, d\omega_i$$

- What about this formulation seems broken?
  - We can't integrate over a dimensionless entity like a vector
  - ω, it turns out, isn't really a vector

BRDFs for Lighting

- ω is a *solid angle*

  "The solid angle, Ω, is the angle in three-dimensional space that an object subtends at a point. It is a measure of how big that object appears to an observer looking from that point."[1]

  - Each ω is a direction and a "slice" from the volume of the hemisphere around the point in question

[1] From http://en.wikipedia.org/wiki/Solid_angle

BRDFs for Lighting

- What if we want to calculate the amount light reflected to a particular output vector from *all possible* input vectors?

$$L(\omega_o) = \int_{\Omega} f(\omega_o, \omega_i)\, L(\omega_i) \cos\theta_i \, d\omega_i$$

  - This integral is over the hemisphere above the point
    - This is a solid angle of 2π
  - Most BRDFs will contain a $1/\pi$ factor because of this

BRDF Properties

- Physically based BRDFs have two important properties:
  - Helmoltz reciprocity:

$$f(\omega_i, \omega_o) = f(\omega_o, \omega_i)$$

    - Also called Helmoltz Stereopsis
    - This is the “bi-directional” part of BRDF
  - Conservation of energy:

$$\forall \omega_i, \int_{\Omega} f(\omega_i, \omega_o) \cos\theta_o \, d\omega_o \leq 1$$

Where do BRDFs come from?

- Measured BRDFs
  - Measure every possible output from every possible output
  - Oregon BRDF Library (and others) have data captured from these instruments available

Measured BRDFs

Image from http://www.merl.com/projects/facescanning/

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Measured BRDFs

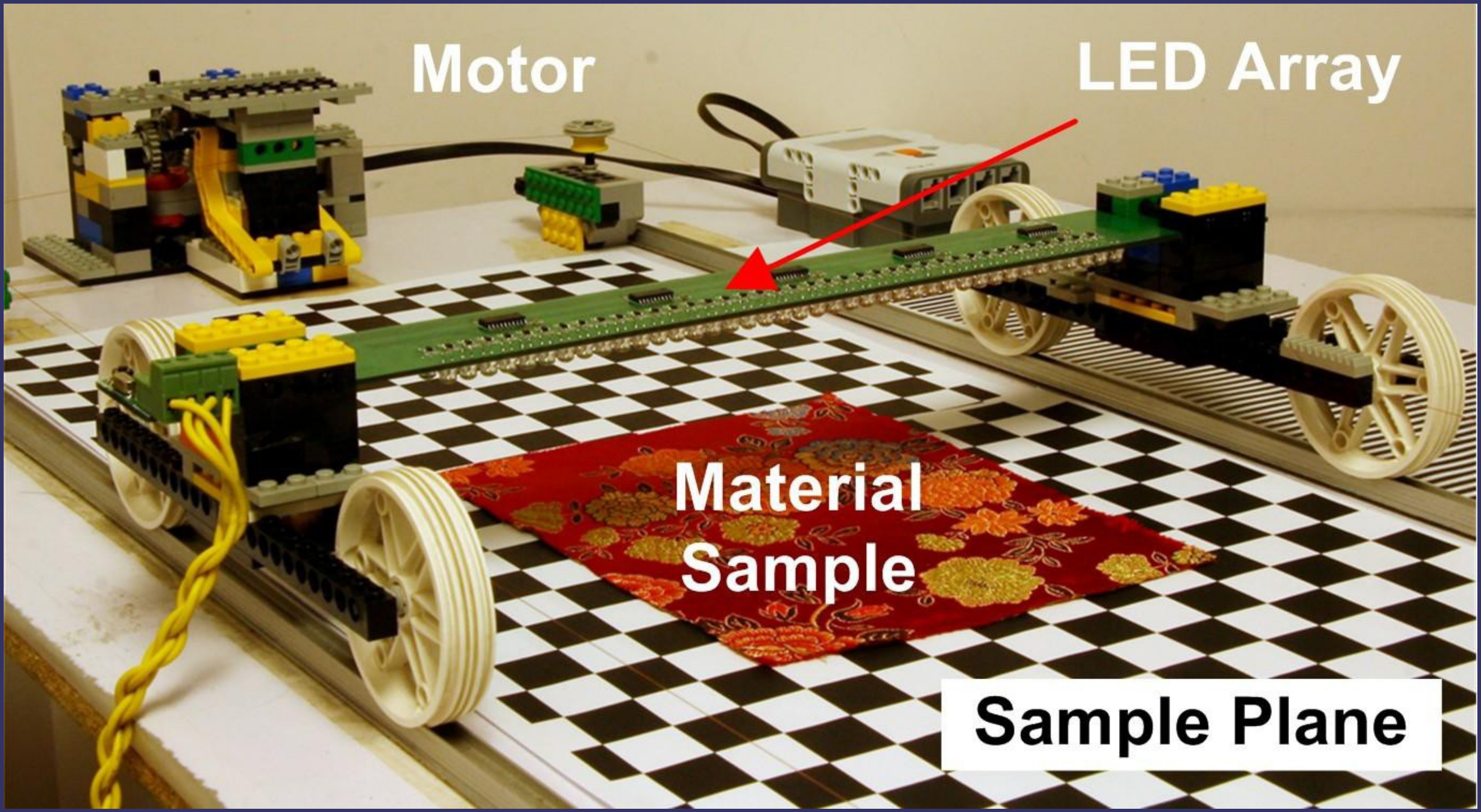

Image from http://www.shuangz.com/projects/aniso/

References

Wang, J., Zhao, S., Tong, X., Snyder, J., and Guo, B. 2008. Modeling anisotropic surface reflectance with example-based microfacet synthesis. In *ACM SIGGRAPH 2008 Papers* (Los Angeles, California, August 11 - 15, 2008). SIGGRAPH '08. ACM, New York, NY, 1-9. http://www.shuangz.com/projects/aniso/

Sample BRDF data sets:

http://www.graphics.cornell.edu/online/measurements/reflectance/index.html

http://www1.cs.columbia.edu/CAVE//software/curet/

http://math.nist.gov/~FHunt/appearance/obl.html

Where do BRDFs come from?

- Measured BRDFs
  - Measure every possible output from every possible output
  - Oregon BRDF Library (and others) have data captured from these instruments available
- Analytical BRDFs
  - Mathematical models used to reproduce observed behavior
  - May be derived from simplified measured data

Cook-Torrance BRDF

- One of the oldest BRDFs used in graphics
  - Developed while Robert Cook was at Lucasfilm and Ken Torrance was at Cornell
  - Published in 1982
  - Based on *micro-facets*

Micro-Facet Primer

- Surfaces are made of numerous infinitesimal subsurfaces that act as perfect mirrors
  - Distribution of the normals of these subsurfaces determines how specular the surface appears
  - Micro-facets can obscure other micro-facets both from the light and from the viewer
- We'll dive deep into both these aspects soon...

Cook-Torrance BRDF

$$f(\omega_o,\omega_i) = K_d f_d + K_s f_s(\omega_o,\omega_i)$$

$$f_d = 1/\pi$$

$$f_s = 1/\pi \frac{F \times D(N \cdot H) \times G(N \cdot \omega_i, N \cdot H, N \cdot \omega_o)}{(N \cdot \omega_i)(N \cdot \omega_o)}$$

- F is the Fresnel factor
- D is the distribution of micro-facet normals
- G is the geometry occlusion factor
- H is the half-vector from the Blinn-Phong lighting equation

Micro-facet Distribution

- Micro-facet normals are random, but follow some distribution function
  - Sometimes call the *normal distribution function* (NDF)
  - Several models exist
  - Cook-Torrance uses the Beckmann Distribution:

$$D(N \cdot H) = \frac{1}{4m^2(N \cdot H)^4} e^{-\left(\frac{1-(N \cdot H)^2}{(N \cdot H)^2 m^2}\right)}$$

    - m is a parameter that controls the smoothness of the surface

Geometry Occlusion Factor

- Represents the decrease in light transmission caused by occlusion of the light or viewer by other micro-facets

$$G(N\cdot\omega_i, N\cdot H, N\cdot\omega_o) = min\left(1, \frac{2(N\cdot H)(N\cdot\omega_o)}{\omega\cdot H}, \frac{2(N\cdot H)(N\cdot\omega_i)}{\omega\cdot H}\right)$$

- Why aren't there any subscripts on w in the denominators?
  - Hint: remember ω_i is L and ω_o is V

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Geometry Occlusion Factor

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- Why aren't there any subscripts on w in the denominators?
  - Hint: remember ω_i is L and ω_o is V
  - H is half way between L and V, so $(H\cdot\omega_i) = (H\cdot\omega_o)$

Cook-Torrance Diffuse Factor

- Cook-Torrance diffuse factor:

$$f_d = 1/\pi$$

- "Typical" diffuse factor:

$$K_d = N \cdot L$$

- Remember how the BRDF is used:

$$L(\omega_o) = f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i$$

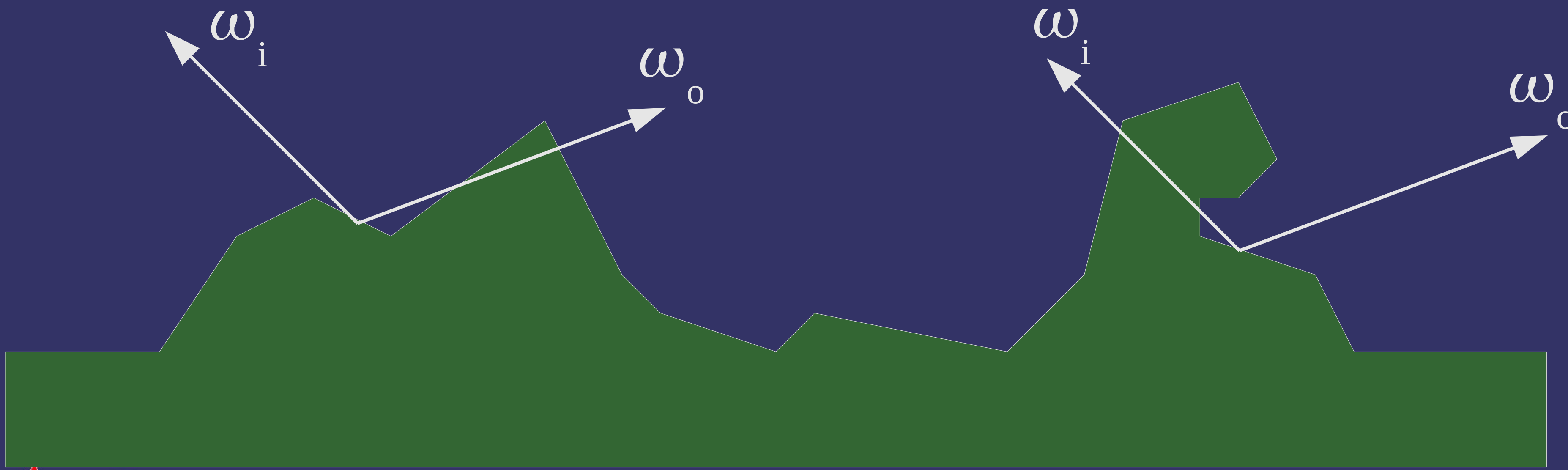
  - We just want to scale the incoming energy by the total angle and let the built in ($N \cdot \omega_i$) do the rest
  - Remember ω_i is L

Micro-Facet Deep Dive

- Surfaces are made of numerous infinitesimal subsurfaces that act as perfect mirrors
  - Each facet only reflects light along the ideal reflection vector
  - Determining the number of visible facets for a given V and L is enough to determine the BRDF

Micro-Facet Deep Dive

- Add two assumptions:
  - Facet normals are distributed randomly according to some distribution function $p(H)$
  - A facet only contributes if it is visible to both V and L



Micro-Facet Deep Dive

- BRDF is determined by:
  - Fresnel term
  - Fraction of micro-facets with $N = H$
  - Fraction of micro-facets visible to both L and V
    - Non-visible to L is often called “shadowing”
    - Non-visible to V is often called “masking”
    - Both can just be called “occlusion”

Normal Distribution

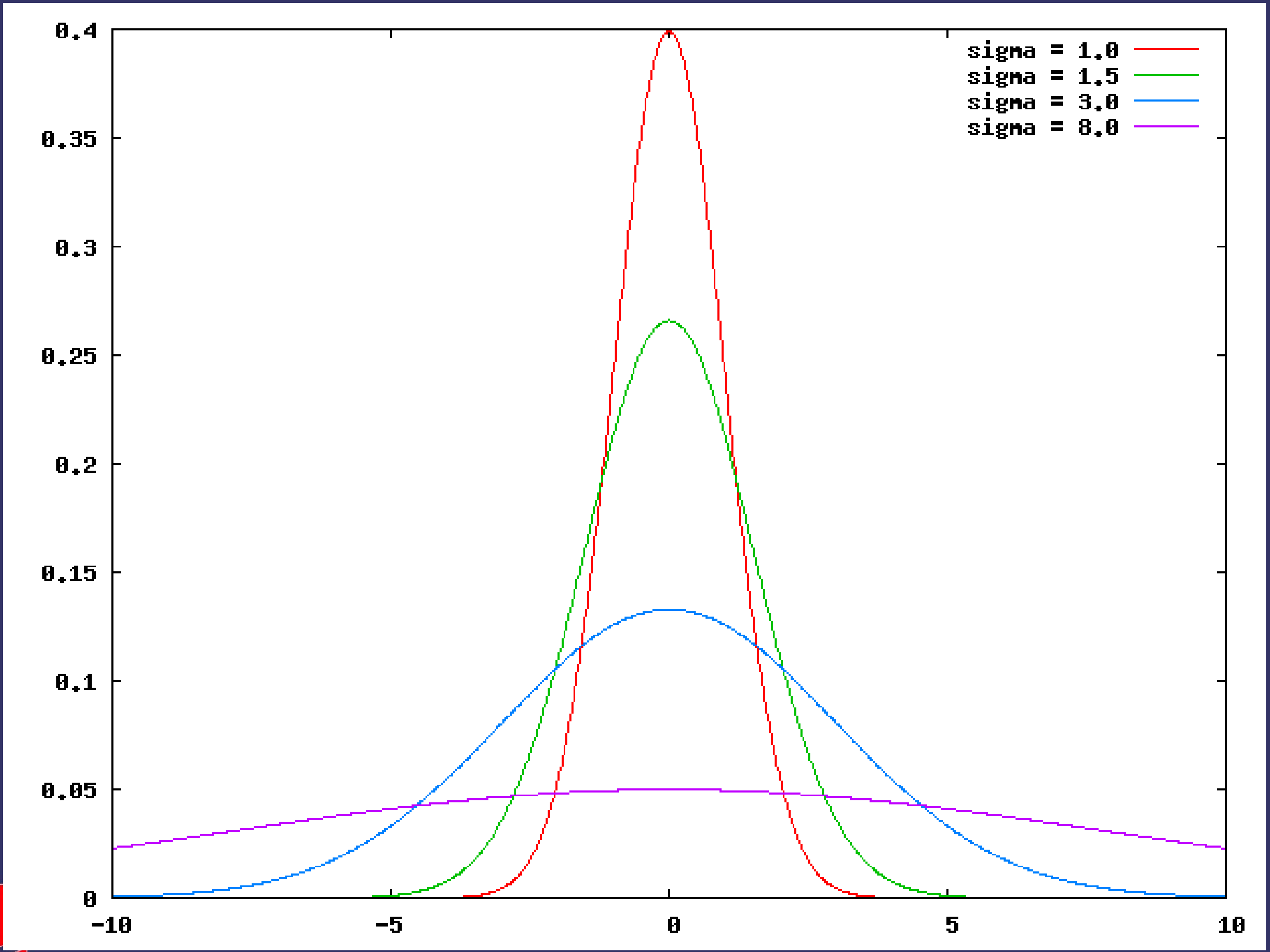
- Given N, determine the fraction of micro-facet normals that point towards H
  - Can use arbitrary function to calculate this probability
    - May be convenient to encode this in a texture
  - Gaussian or *standard normal distribution* function seems like a good choice
    - The more different the H is from N, the lower the probability

Gaussian Distribution

$$P(\theta)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\left(\frac{\theta^2}{2\sigma^2}\right)}$$

σ is the standard deviation

Gaussian Distribution



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Gaussian Distribution

$$P(\theta)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\left(\frac{\theta^2}{2\sigma^2}\right)}$$

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- Looking at the graph, why is this distribution unsuitable?

Gaussian Distribution

$$P(\theta)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\left(\frac{\theta^2}{2\sigma^2}\right)}$$

σ is the standard deviation

- Looking at the graph, why is this distribution unsuitable?
  - As σ increases, the effective range increases to ∞
  - Distribution is based on θ, but we only know cos(θ)

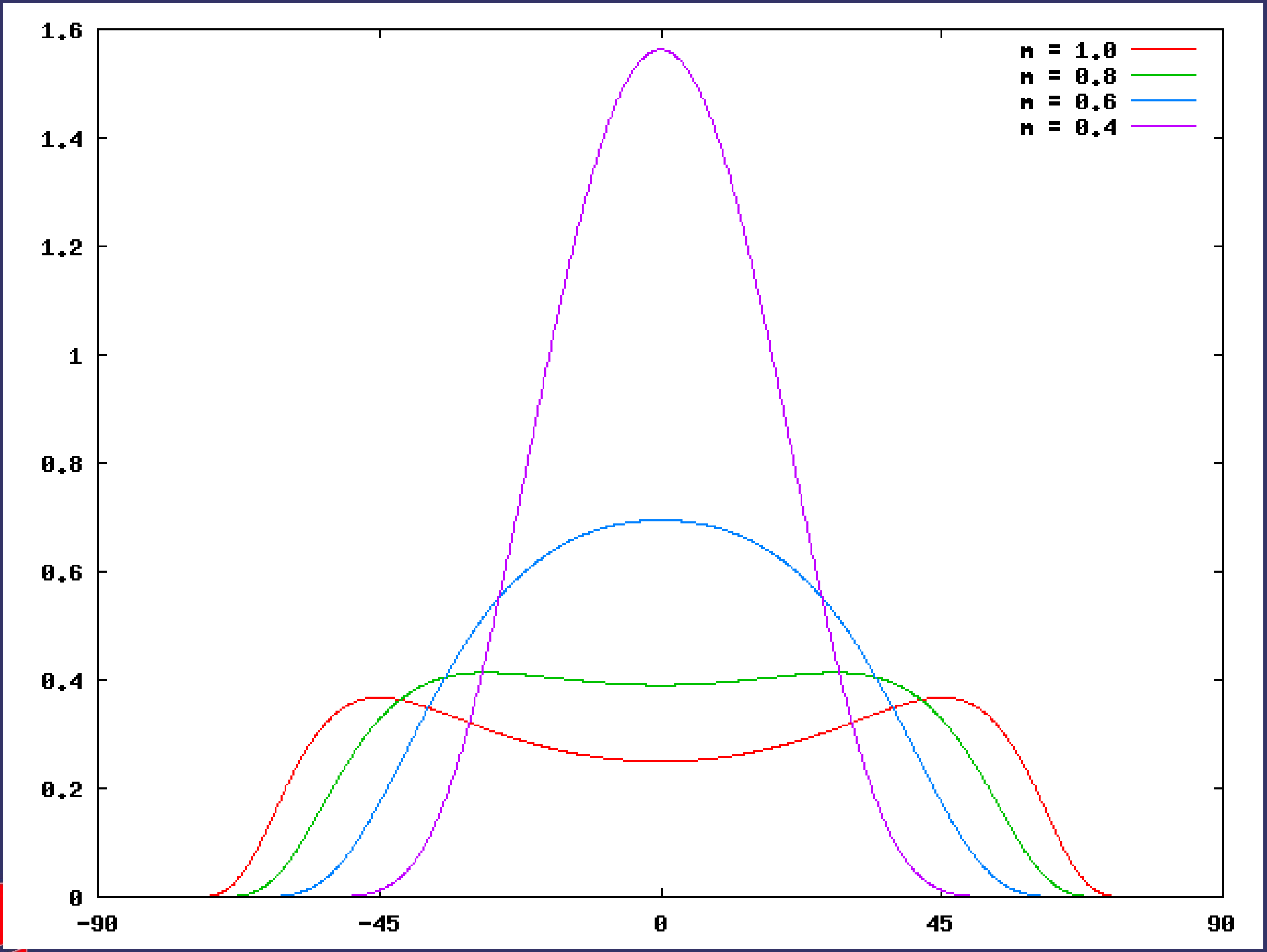
Beckmann Distribution

$$P(\theta) = \frac{1}{4m^2\cos^4\theta} e^{-\left(\frac{\tan^2\theta}{m^2}\right)}$$

m is average slope of the surface micro-facets

- Physically based model of rough surfaces
  - Based on Beckmann's research in the early 60s
- All calculations are based on cos(θ)!
  - Remember: $\tan^2(\theta)$ is $(1 - \cos^2(\theta)) / \cos^2(\theta)$

Beckmann Distribution



Micro-facet Occlusion

- Determine the probability of a facet being visible to the light and to the viewer
  - Use one probability function, $P_v(\theta)$, for the probability of visibility to either L or V
  - Assume that visibility and orientation are uncorrelated

Micro-facet Occlusion

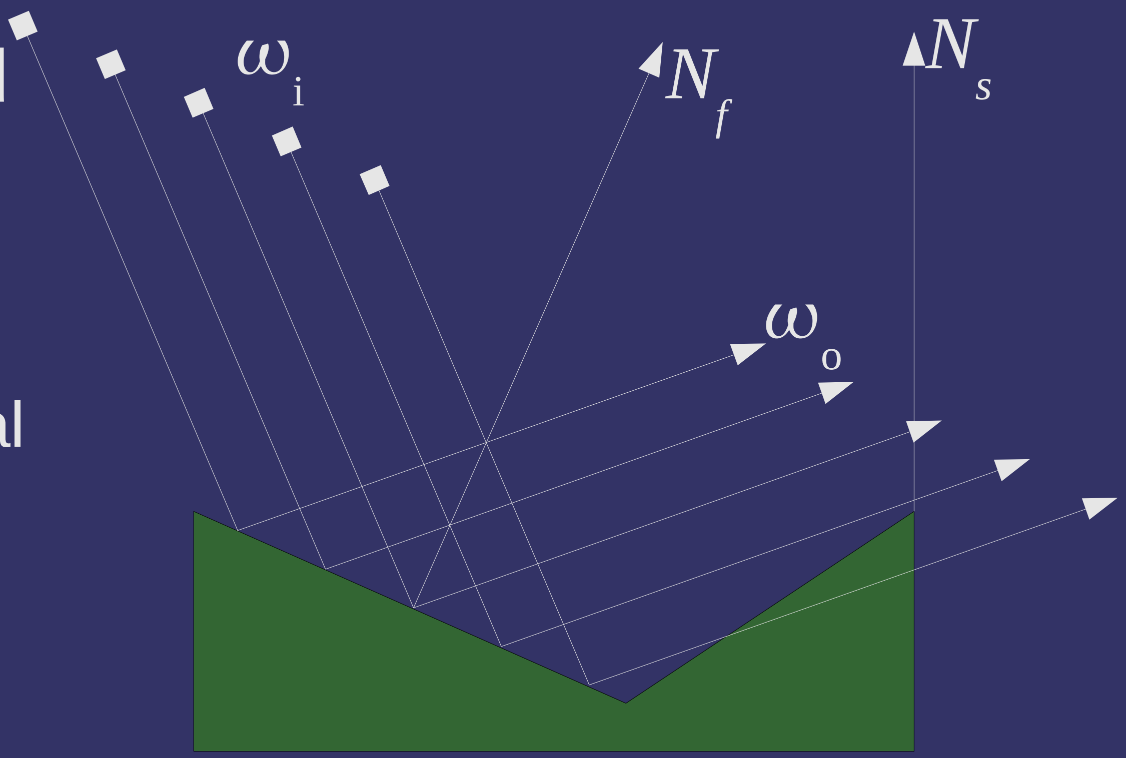
- If $P_v(\theta_V)$ and $P_v(\theta_L)$ are known how do we compute $P_v(\theta_V \cap \theta_L)$?

Micro-facet Occlusion

- If $P_v(\theta_V)$ and $P_v(\theta_L)$ are known how do we compute $P_v(\theta_V \cap \theta_L)$?
  - Generating a new probability function from dependent probability functions is a difficult problem in general
  - Multiplying the two probabilities underestimates the visibility
  - Cook and Torrance suggest taking the smaller value
  - Other methods exist...this weeks reading contains one

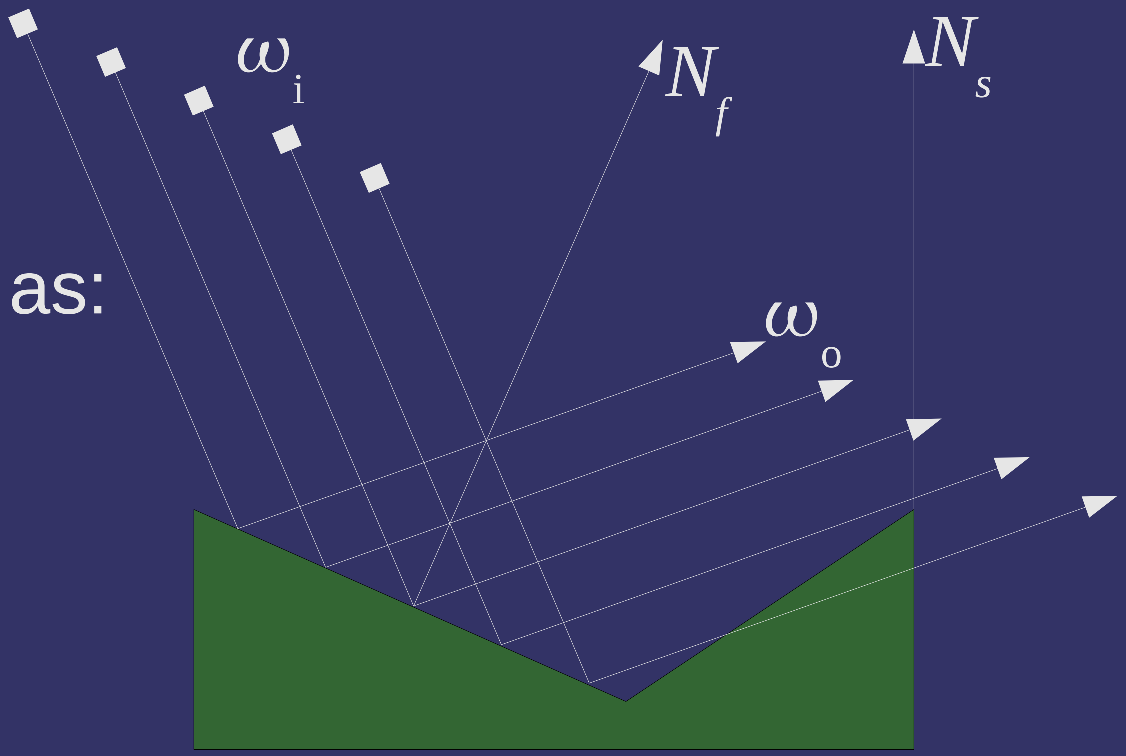
Micro-facet Occlusion

- How do we estimate $P_v(\theta)$?
  - Clearly ω_i, ω_o, N_f, and N_s are involved
    - N_f is the facet normal
    - N_s is the surface normal



Micro-facet Occlusion

- Observations:
  - Occlusion increases as:
    - $(N_f \cdot N_s) \rightarrow 0$
    - $(\omega \cdot N_s) \rightarrow 0$
  - Occlusion decreases as:
    - $(\omega \cdot N_f) \rightarrow 0$



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Micro-facet Occlusion

- Cook-Torrance uses:

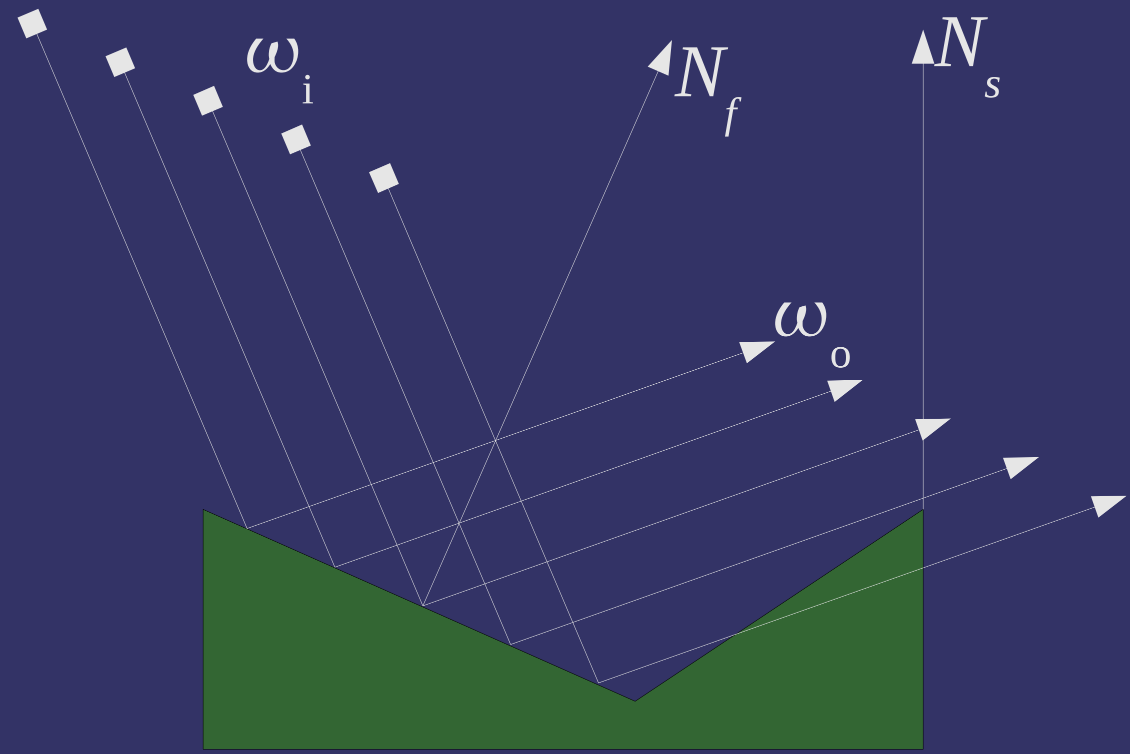
$$P_v(\theta) = \frac{2(N_s \cdot N_f)(N_s \cdot \omega)}{\omega \cdot N_f}$$

- What is N_f?
  - H!

$$G_V = \frac{2(N \cdot H)(N \cdot V)}{V \cdot H}$$

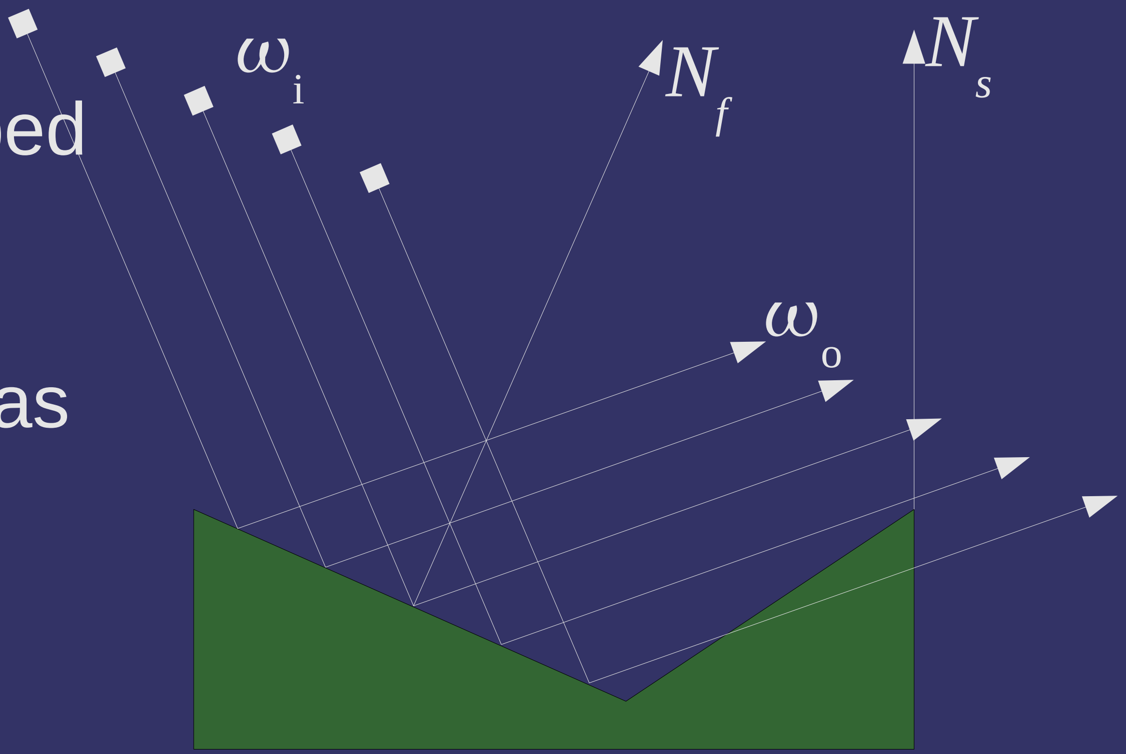
$$G_L = \frac{2(N \cdot H)(N \cdot L)}{L \cdot H}$$

$$L \cdot H = V \cdot H$$



Micro-facet Occlusion

- This turns out to be a poor model
  - Real surfaces aren't made of long, V-shaped channels
  - This reading for next week addresses this as well



References

http://wiki.gamedev.net/index.php/D3DBook:(Lighting)_Cook-Torrance

Philip Dutré. “Global Illumination Compendium.” Computer Graphics, Department of Computer Science Katholieke Universiteit Leuven. 2003. http://www.cs.kuleuven.ac.be/~phil/GI/

References that we should have had *last week* for pre-filtered reflection maps:

Michael Ashikhmin and Abhijeet Ghosh. “Simple Blurry Reflections with Environment Maps.” Journal of Graphics Tools, 7(4): 3-8, 2002. http://people.ict.usc.edu/~ghosh/papers.html

R. Ramamoorthi and P. Hanraham. “An Efficient Representation for Irradiance Environment Maps.” In *Proceedings of SIGGRAPH 2001, Computer Graphics Proceedings*, Annual Conference Series, edited by E. Fiume, pp. 497—500, Reading, MA: Addison-Wesley, 2001. http://www-graphics.stanford.edu/papers/envmap/

Reading

Ashikmin, Michael and Premože, Simon and Shirley, Peter, "A microfacet-based BRDF generator." In *SIGGRAPH '00: Proceedings of the 27th Annual Conference on Computer Graphics and Interactive Techniques*, pages 65–74. ACM Press/Addison-Wesley Publishing Co., 2000. http://www.cs.utah.edu/vissim/papers/facets/

Next week...

- More BRDFs
  - Anisotropic reflection
    - Ward BRDF
    - Ashikhmin BRDF
  - Metals
    - How do metals “reflect” light?
    - Lafortune BRDF

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